

# Addition and Subtraction Within 10 <u>Math in Focus</u>

Unit 1 Curriculum Guide September 9<sup>th</sup>- November 13<sup>th</sup>, 2019



ORANGE PUBLIC SCHOOLS

OFFICE OF CURRICULUM AND INSTRUCTION

OFFICE OF MATHEMATICS

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## First Grade Unit I Chapter 1-4 & Eureka Math Module 1 (TOPIC E,F,G,H,I)

#### In this Unit Students will:

#### 1.0A.1-8

- > Solve addition and subtraction situations involving:
  - Adding to,
  - Taking From,
  - Putting Together,
  - Taking Apart, and
  - Comparing situations.
- > Apply the following problem solving strategies
  - Use of objects and/or drawings
  - Counting On
  - Making Ten
  - Decomposing Numbers
  - Properties of Operations
  - Relationship between Addition and Subtraction

#### 1.NBT.1

- > Count to 120 starting at any number less than 120
- > Reade and write numerals
- > Represent number of objects with a written numeral

#### **Mathematical Practices**

- ➤ Make sense of persevere in solving them.
- > Reason abstractly and quantitatively.
- ➤ Construct viable arguments and critique the reasoning of others.
- ➤ Model with mathematics.
- Use appropriate mathematical tools.
- > Attend to precision.
- ➤ Look for and make use of structure.
- ➤ Look for and express regularity in repeated reasoning.

Activities	NJSLS	Notes
Math In Focus Ch.1 Opener Introduction to <b>Math Workstations,</b> and Notebook Expectations	K.CC.4, K.CC.6	Routines/Procedures Ask 3 Then Me; Math Talk Moves;
Math In Focus Ch. 1 Lesson 1 Day 1 Counting to 10	1.NBT.1	
Math In Focus Ch. 1 Lesson 1 Day 2 Counting to 10	1.NBT.1	
Math In Focus Ch. 1 Lesson 3 Making Number Patterns	1.NBT.1	Math In Focus:
<b>Math In Focus Ch. 1</b> Put On Your Thinking Cap Chapter Wrap Up & Chapter Review		Thinkcentral.com Access Assessments Pre-Test Chapter Poview
Math In Focus Ch. 1 Test Math In Focus Ch. 1 Performance Task		Chapter Review Test Prep Performance Tasks
Math In Focus Ch.2 Opener Introduction to Math Workstations		
Math In Focus Ch.2 Lesson 1 Day 1 Number Bonds	1.0A.3	
Math In Focus Ch.2 Lesson 1 Day 2 Number Bonds	1.0A.3	
Math In Focus Ch. 2 Put On Your Thinking Cap Chapter Wrap Up & Chapter Review	1.0A.6	
Math In Focus Ch. 2 Test Math In Focus Ch. 2 Performance Task		
<b>Math In Focus Ch.3 Opener</b> Math Workstations		
Math in Focus Ch. 3 lesson 1 Day 1 Ways to add	1.0A.1, 1.0A.3, 1.0A.5, 1.0A.7	
Math in Focus Ch. 3 lesson 1 Day 2 Ways to add	1.0A.1, 1.0A.3, 1.0A.5, 1.0A.7	
Math in Focus Ch. 3 lesson 1 Day 3 Ways to add	1.0A.1, 1.0A.3, 1.0A.5, 1.0A.7	
Math in Focus Ch. 3 lesson 2 Making Addition Stories	1.OA.6, 1.OA.7	
Math in Focus Ch.3 lesson 3	1.0A.1, 1.0A.6,	

Real World Problems- Addition	1.0A.7, 1.0A.8
Math In Focus Ch. 3 Put On Your Thinking Cap	1.0A.7, 1.0A.8
Chapter Wrap Up & Chapter Review	
Math in Focus Ch. 3 Test Math In Focus Ch. 3 Performance Task	
Math In Focus Ch.4 Opener Math Workstations	
Math in Focus Ch. 4 lesson 1 Day 1 Ways to Subtract	1.0A.1, 1.0A.4, 1.0A.5, 1.0A.7
Math in Focus Ch. 4 lesson 1 Day 2 Ways to Subtract	1.0A.1, 1.0A.4, 1.0A.5, 1.0A.7
Math in Focus Ch. 4 lesson 2 Making Subtraction Stories	1.0A.4, 1.0A.6, 1.0A.7
Math in Focus Ch. 4 lesson 3 Real World Problems- Subtraction	1.0A.1, 1.0A.4, 1.0A.7, 1.0A.8
Math in Focus Ch. 4 lesson 4 Making Fact Families	1.0A.1, 1.0A.4, 1.0A.7
Math in Focus Ch. 4 lesson 4 Making Fact Families	1.0A.1, 1.0A.4, 1.0A.7
Math In Focus Ch. 4 Put On Your Thinking Cap Chapter Wrap Up & Chapter Review	1.0A.4, 1.0A.7
Math in Focus Ch. 4 Test Prep Math In Focus Ch. 4 Performance Task	

## Eureka Math Module 1: Sums and Differences to 10 TOPIC E,F,G,H,I

	Lesson 17	Understand the meaning of the equal sign by pairing equivalent expressions and constructing true number sentences. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
Topic E:		
The Commutative Property of Addition and the	Lesson 18	Understand the meaning of the equal sign by pairing equivalent expressions and constructing true number sentences. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
Equal Sign	Lesson 19	Represent the same story scenario with addends repositioned (the commutative property). <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 20	Apply the commutative property to count on from a larger addend. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>

	Lesson 21	Visualize and solve doubles and doubles plus 1 with 5-group cards. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
<b>Topic F:</b> Development of Addition	Lesson 22	Look for and make use of repeated reasoning on the addition chart by solving and analyzing problems with common addends. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
Fluency within 10	Lesson 23	Look for and make use of structure on the addition chart by looking for and coloring problems with the same total. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 24	Practice to build fluency with facts to 10. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
Topic G:	Lesson 25	Solve add to with change unknown math stories with addition and relate to subtraction. Model with materials and write corresponding number sentences.
Subtraction as an Unknown Addend		https://www.youtube.com/watch?v
& Topic H:	Lesson 30	Solve add to with change unknown math stories with drawings, relating addition and subtraction. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
Subtraction Word Problems	Lesson 28/29	Solve take from with result unknown math stories with math drawings, true number sentences and statements, using horizontal marks to cross off what is taken away.
		https://www.youtube.com/watch?v  Solve take apart with addend unknown math stories with math
		drawings, equations, and statements, circling the known part to find the unknown.
		https://www.youtube.com/watch?v
	Lesson 31	Solve take from with change unknown math stories with drawings. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 32	Solve put together/take apart with addend unknown math stories. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 33	Model 0 less and 1 less pictorially and as subtraction number sentences.
		https://www.youtube.com/watch?v

<b>Topic I:</b> Decomposition	Lesson 34	Model $n-n$ and $n-(n-1)$ pictorially and as subtraction sentences. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
Strategies for Subtraction	Lesson 35	Relate subtraction facts involving fives and doubles to corresponding decompositions. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 36	Relate subtraction from ten to corresponding decompositions. <a href="https://www.youtube.com/watch?v">https://www.youtube.com/watch?v</a>
	Lesson 37	Relate subtraction from nine to corresponding decompositions. https://www.youtube.com/watch?v

<b>Modifications</b>			
Special Education/ 504:	English Language Learners:		
-Adhere to all modifications and health concerns stated in each IEP.  -Give students a Menu, allowing students to pick assignments from different levels based on difficulty.  -Accommodate Instructional Strategies: reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), handouts, definition list with visuals, extended time -Allow students to demonstrate understanding of a problem by drawing the picture of the answer and then explaining the reasoning orally and/or in writing, such as Read-Draw-Write  -Provide breaks between tasks, use positive reinforcement, use proximity  -Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum by using manipulatives  -Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 17-18)	- Use manipulatives to promote conceptual understanding and enhance vocabulary usage - Provide graphic representations, gestures, drawings, equations, realia, and pictures during all segments of instruction - During i-Ready lessons, click on "Español" to hear specific words in Spanish - Utilize graphic organizers which are concrete, pictorial ways of constructing knowledge and organizing information - Use sentence frames and questioning strategies so that students will explain their thinking/ process of how to solve word problems - Utilize program translations (if available) for L1/ L2 students - Reword questions in simpler language - Make use of the ELL Mathematical Language Routines (click here for additional information) -Scaffolding instruction for ELL Learners - Common Core Approach to Differentiate Instruction: Students with Disabilities (pg 16-17)		
Gifted and Talented:	Students at Risk for Failure:		
<ul> <li>Elevated contextual complexity</li> <li>Inquiry based or open ended assignments and projects</li> <li>More time to study concepts with greater depth</li> <li>Promote the synthesis of concepts and making real world connections</li> <li>Provide students with enrichment practice that are imbedded in the curriculum such as: <ul> <li>Application / Conceptual Development</li> <li>Are you ready for more?</li> </ul> </li> <li>Common Core Approach to Differentiate Instruction: Students with Disabilities (pg. 20)</li> <li>Provide opportunities for math competitions</li> <li>Alternative instruction pathways available</li> </ul>	- Assure students have experiences that are on the Concrete- Pictorial- Abstract spectrum - Modify Instructional Strategies, reading aloud text, graphic organizers, one-on-one instruction, class website (Google Classroom), inclusion of more visuals and manipulatives, Field Trips, Google Expeditions, Peer Support, one on one instruction - Assure constant parental/ guardian contact throughout the year with successes/ challenges - Provide academic contracts to students and guardians - Create an interactive notebook with samples, key vocabulary words, student goals/ objectives Always plan to address students at risk in your learning tasks, instructions, and directions. Try to anticipate where the needs will be and then address them prior to lessonsCommon Core Approach to Differentiate Instruction: Students with Disabilities (pg 19)		

#### 21st Century Life and Career Skills:

Career Ready Practices describe the career-ready skills that all educators in all content areas should seek to develop in their students. They are practices that have been linked to increase college, career, and life success. Career Ready Practices should be taught and reinforced in all career exploration and preparation programs with increasingly higher levels of complexity and expectation as a student advances through a program of study.

https://www.state.nj.us/education/cccs/2014/career/9.pdf

- **CRP1**. Act as a responsible and contributing citizen and employee.
- CRP2. Apply appropriate academic and technical skills.
- CRP3. Attend to personal health and financial well-being.
- CRP4. Communicate clearly and effectively and with reason.
- CRP5. Consider the environmental, social and economic impacts of decisions.
- CRP6. Demonstrate creativity and innovation.

- **CRP7**. Employ valid and reliable research strategies.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- **CRP9**. Model integrity, ethical leadership and effective management.
- **CRP10**. Plan education and career paths aligned to personal goals.
- **CRP11**. Use technology to enhance productivity.
- CRP12. Work productively in teams while using cultural global competence.

Students are given an opportunity to communicate with peers effectively, clearly, and with the use of technical language. They are encouraged to reason through experiences that promote critical thinking and emphasize the importance of perseverance. Students are exposed to various mediums of technology, such as digital learning, calculators, and educational websites.

#### **Technology Standards:**

All students will be prepared to meet the challenge of a dynamic global society in which they participate, contribute, achieve, and flourish through universal access to people, information, and ideas.

https://www.state.nj.us/education/cccs/2014/tech/

#### 8.1 Educational Technology:

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and to create and communicate knowledge.

- A. **Technology Operations and Concepts:** Students demonstrate a sound understanding of technology concepts, systems and operations.
- B. **Creativity and Innovation:** Students demonstrate creative thinking, construct knowledge and develop innovative products and process using technology.
- C. Communication and Collaboration: Students use digital media and environments to communicate and work collaboratively, including at a distance, to support individual learning and contribute to the learning of others.
- D. **Digital Citizenship:** Students understand human, cultural, and societal issues related to technology and practice legal and ethical behavior.
- E. **Research and Information Fluency:** Students apply digital tools to gather, evaluate, and use of information.
- F. Critical thinking, problem solving, and decision making: Students use critical thinking skills to plan and conduct research, manage projects, solve problems, and make informed decisions using appropriate digital tools and resources.

## 8.2 Technology Education, Engineering, Design, and Computational Thinking Programming:

All students will develop an understanding of the nature and impact of technology, engineering, technological design, computational thinking and the designed world as they relate to the individual, global society, and the environment.

- A. The Nature of Technology: Creativity and Innovation- Technology systems impact every aspect of the world in which we live.
- B. Technology and Society: Knowledge and understanding of human, cultural, and societal values are fundamental when designing technological systems and products in the global society.
- C. **Design:** The design process is a systematic approach to solving problems.
- D. Abilities in a Technological World: The designed world in a product of a design process that provides the means to convert resources into products and systems.
- E. Computational Thinking: Programming-Computational thinking builds and enhances problem solving, allowing students to move beyond using knowledge to creating knowledge.

Interdisciplinary Connections:			
English Language Arts:			
RF.1.4	Read with sufficient accuracy and fluency to support comprehension.		
W.1.2	Write informative/explanatory texts in which they name a topic, supply some facts about the topic, and provide some sense of closure.		
SL.1.1	Participate in collaborative conversations with diverse partners about <i>grade 1 topics and texts</i> with peers and adults in small and larger groups.		

New Jersey Student Learning Standards: Operations and Algebraic Thinking			
1.OA.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.		

First grade students extend their experiences in Kindergarten by working with numbers to 20 to solve a new type of problem situation: Compare (See **Table 1** at end of document for examples of all problem types). In a Compare situation, two amounts are compared to find "How many more" or "How many less".

Problem Type: Compare			
Difference Unknown: Bigger Unknown:		Smaller Unknown:	
"How many more?" version.	"More" version suggests operation.	Version with "more"	
Lucy has 7 apples. Julie as 9	Julie has 2 more apples than Lucy.		
apples. How many more apples	Lucy has 7 apples. How many	Mastery expected in Second Grade	
does Julie have than Lucy?	apples does Julie have?		
"How many fewer?" version	Bigger Unknown:	Smaller Unknown:	
Lucy as 7 apples. Julie has 9	Version with "fewer"	"Fewer" version suggests operation.	
apples. How many fewer apples		Lucy has 2 fewer apples than Julie.	
does Lucy have than Julie?	Mastery expected in	Julie has 9 apples. How many apples	
7+ 🗖 = 9	Second Grade	does Lucy have?	
9 – 7 = □			

Table 1 Common addition and subtraction situations<sup>1</sup>

	Result Unknown	Change Unknown	Start Unknown
	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now?	Two bunnies were sitting on the grass. Some more bunnies hopped there. Then there were five bunnies. How many bunnies hopped over	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were five bunnies. How many bunnies were on the grass before?
Add to	2+3=?	to the first two? 2+?=5	?+3=5
	(K)	(1 <sup>st</sup> )	One-Step Problem (2 <sup>nd</sup> )
	Five apples were on the table. I ate two	Five apples were on the table. I ate some	Some apples were on the table. I ate two apples.
Take from	apples. How many apples are on the table now?	apples. Then there were three apples. How	Then there were three apples. How many apples were on the table before? $?-2=3$
1 ake Irom	now? 5-2=?	many apples did I eat? 5-?=3	on the table before $i - 2 = 3$
	3-2-? (K)	(1 <sup>st</sup> )	One-Step Problem (2 <sup>nd</sup> )
	(11)	(2)	One Step 11 south
	Total Unknown	Addend Unknown	Both Addends Unknown <sup>2</sup>
	Three red apples and two green apples are on	Five apples are on the table. Three are red and	Grandma has five flowers. How many can she put in
	the table. How many apples are on the table?	the rest are green. How many apples are	her red vase and how many in her blue vase?
Put Together/	3+2=?	green?	5 = 0 + 5, 5 = 5 + 0
Take Apart <sup>3</sup>		3+?=5,5-3=?	5 = 1 + 4, 5 = 4 + 1
	(K)	(K)	5 = 2 + 3, 5 = 3 + 2 (1 <sup>st</sup> )
	(A)	(A)	(1)
	Difference Unknown	Bigger Unknown	Smaller Unknown
	("How many more?" version):	(Version with "more"):	(Version with "more"):
	Lucy has two apples. Julie has five apples.	Julie has three more apples than Lucy. Lucy	Julie has 3 more apples than Lucy. Julie has five
	How many more apples does Julie have than Lucy?	has two apples. How many apples does Julie have?	apples. How many apples does Lucy have?
	an.		5-3=? ?+3=5
Compare <sup>4</sup>	(1 <sup>1</sup> )	One-Step Problem (1 <sup>st</sup> )	One-Step Problem (2 <sup>nd</sup> )
	("How many fewer?" version):	(Version with "fewer"):	(Version with "fewer"):
	Lucy has two apples. Julie has five apples.	Lucy has 3 fewer apples than Julie. Lucy has	Lucy has three fewer apples than Julie. Julie has five
	How many fewer apples does Lucy have than Julie?	two apples. How many apples does Julie have? $2+3=?$ , $3+2=?$	apples. How many apples does Lucy have?
	2+?=5, 5-2=?		
	(1 <sup>st</sup> )	One-Step Problem (2 <sup>nd</sup> )	One-Step Problem (1 <sup>st</sup> )

K: Problem types to be mastered by the end of the Kindergarten year.

1st: Problem types to be mastered by the end of the First Grade year, including problem types from the previous year(s). However, First Grade students should have experiences with all 12 problem types.

2nd: Problem types to be mastered by the end of the Second Grade year, including problem types from the previous year(s).

#### New Jersey Student Learning Standards: Operations and Algebraic Thinking

1.0A.2

Solve word problems that call for addition of three whole numbers whose sum is less than or equal to 20.

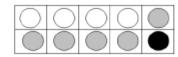
First Grade students solve multi-step word problems by adding (joining) three numbers whose sum is less than or equal to 20, using a variety of mathematical representations.

#### Example:

Mrs. Smith has 4 oatmeal raisin cookies, 5 chocolate chip cookies, and 6 gingerbread cookies. How many cookies does Mrs. Smith have?

#### Student A:

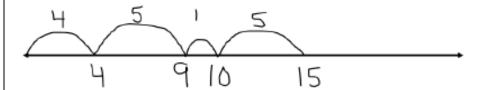
I put 4 counters on the Ten Frame for the oatmeal raisin cookies. Then, I put 5 different color counters on the ten frame for the chocolate chip cookies. Then, I put another 6 color counters out for the gingerbread cookies. Only one of the gingerbread cookies fit, so I had 5 leftover. Ten and five more makes 15 cookies. Mrs. Smith has 15 cookies.





#### Student B:

I used a number line. First I jumped to 4, and then I jumped 5 more. That's 9. I broke up 6 into 1 and 5 so I could jump 1 to make 10. Then, I jumped 5 more and got 15. Mrs. Smith has 15 cookies.



#### Student C:

I wrote:  $4 + 5 + 6 = \square$  I know that 4 and 6 equals 10, so the oatmeal raisin and gingerbread equals 10 cookies.

Then I added the 5 chocolate chip cookies. 10 and 5 is 15. So, Mrs. Smith has 15 cookies.

New Jersey Student Lear	New Jersey Student Learning Standards: Operations and Algebraic Thinking				
1.0A.3	Apply properties of operations as strategies to add and subtract. Examples: If $8 + 3 = 11$ is known, then $3 + 8 = 11$ is also known. <b>(Commutative property of addition.)</b> To add $2 + 6 + 4$ , the second two numbers can be added to make a ten, so $2 + 6 + 4 = 2 + 10 = 12$ . <b>(Associative property of addition.)</b> Students need not use formal terms for these properties.				

Elementary students often believe that there are hundreds of isolated addition and subtraction facts to be mastered. However, when students understand the commutative and associative properties, they are able to use relationships between and among numbers to solve problems. First Grade students apply properties of operations as strategies to add and subtract. Students do not use the formal terms "commutative" and associative". Rather, they use the understandings of the commutative and associative property to solve problems.

Commutative Property of Addition	Associative Property of Addition
The order of the addends does not change	The grouping of the 3 or more addends does not affect the
the sum.	sum.
For example, if 8 + 2 = 10 is known, then 2 + 8 = 10 is also known.	For example, when adding 2 + 6 + 4, the sum from adding the first two numbers first (2 + 6) and then the third number (4) is the same as if the second and third numbers are added first (6 + 4) and then the first number (2). The student may note that 6+4 equals 10 and add those two numbers first before adding 2. Regardless of the order, the sum remains 12.

Students use mathematical tools and representations (e.g., cubes, counters, number balance, number line, 100 chart) to model these ideas.

**Commutative Property Examples: Cubes** 

A student uses 2 colors of cubes to make as many different combinations of 8 as possible.

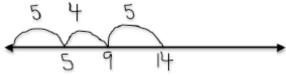
When recording the combinations, the student records that 3 green cubes and 5 blue cubes

equals 8 cubes in all. In addition, the student notices that 5 green cubes and 3 blue cubes also equals 8 cubes.

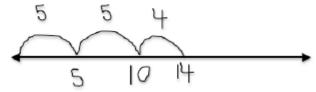
#### **Associative Property Examples:**

Number Line:  $\square = 5 + 4 + 5$ 

**Student A**: First I jumped to 5. Then, I jumped 4 more, so I landed on 9. Then I jumped 5 more and landed on 14.



**Student B**: I got 14, too, but I did it a different way. First I jumped to 5. Then, I jumped 5 again. That's 10. Then, I jumped 4 more. See, 14!



Mental Math: There are 9 red jelly beans, 7 green jelly beans, and 3 black jelly beans. How many jelly beans are there in all? Student: "I know that 7 + 3 is 10. And 10 and 9 is 19. There are 19 jelly beans."

#### 1.0A.4

Understand subtraction as an unknown-addend problem

First Graders often find subtraction facts more difficult to learn than addition facts. By understanding the relationship between addition and subtraction, First Graders are able to use various strategies described below to solve subtraction problems.

#### For Sums to 10

#### \*Think-Addition:

Think-Addition uses known addition facts to solve for the unknown part or quantity within a problem. When students use this strategy, they think, "What goes with this part to make the total?" The think-addition strategy is particularly helpful for subtraction facts with sums of 10 or less and can be used for sixty-four of the 100 subtraction facts. Therefore, in order for think-addition to be an effective strategy, students must have mastered addition facts first.

For example, when working with the problem  $9 - 5 = \square$ , First Graders think "Five and what makes nine?", rather than relying on a counting approach in which the student counts 9, counts off 5, and then counts what's left. When subtraction is presented in a way that encourages students to think using addition, they use known addition facts to solve a problem.

Example:  $10 - 2 = \square$ 

Student: "2 and what make 10? I know that 8 and 2 make 10. So, 10 - 2 = 8."

#### For Sums Greater than 10

The 36 facts that have sums greater than 10 are often considered the most difficult for students to master. Many students will solve these particular facts with Think-Addition (described above), while other students may use other strategies described below, depending on the fact. Regardless of the strategy used, all strategies focus on the relationship between addition and subtraction and often use 10 as a benchmark number.

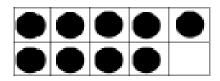
#### \*Build Up Through 10:

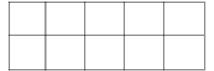
This strategy is particularly helpful when one of the numbers to be subtracted is 8 or 9. Using 10 as a bridge, either 1 or 2 are added to make 10, and then the remaining amount is added for the final sum.

Example:  $15 - 9 = \square$ 

**Student A:** "I'll start with 9. I need one more to make 10. Then, I need 5 more to make 15. That's 1 and 5- so it's 6. 15 0 9 = 6."

**Student B:** "I put 9 counters on the 10 frame. Just looking at it I can tell that I need 1 more to get to 10. Then I need 5 more to get to 15. So, I need 6 counters."





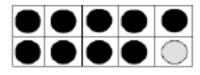
*Back	Down	Thro	ugh	10

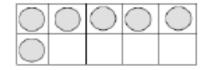
This strategy uses take-away and 10 as a bridge. Students take away an amount to make 10, and then take away the rest. It is helpful for facts where the ones digit of the two-digit number is close to the number being subtracted.

Example:  $16 - 7 = \square$ 

**Student A**: "I'll start with 16 and take off 6. That makes 10. I'll take one more off and that makes 9. 16 - 7 = 9."

Student B: "I used 16 counters to fill one ten frame completely and most of the other one. Then, I can take these 6 off from the 2nd ten frame. Then, I'll take one more from the first ten frame. That leaves 9 on the ten frame."





1.0A.5

Relate counting to addition and subtraction

When solving addition and subtraction problems to 20, First Graders often use counting strategies, such as counting all, counting on, and counting back, before fully developing the essential strategy of using 10 as a benchmark number. Once students have developed counting strategies to solve addition and subtraction problems, it is very important to move students toward strategies that focus on composing and decomposing number using ten as a benchmark number, as discussed in 1.0A.6, particularly since counting becomes a hindrance when working with larger numbers. By the end of First Grade, students are expected to use the strategy of 10 to solve problems.

**Counting All**: Students count all objects to determine the total amount.

**Counting On & Counting Back:** Students hold a "start number" in their head and count on/back from that number.

Example:  $15 + 2 = \Box$ 

#### Counting All

The student counts out fifteen counters. The student adds two more counters. The student then counts all of the counters starting at 1 (1, 2, 3, 4,...14, 15, 16, 17) to find the total amount.

Example:  $12 - 3 = \square$ 

#### Counting All

The student counts out twelve counters. The student then removes 3 of them. To determine the final amount, the student counts each one (1, 2, 3, 4, 5, 6, 7, 8, 9) to find out the final amount.

#### Counting On

Holding 15 in her head, the student holds up one finger and says 16, then holds up another finger and says 17. The student knows that 15 + 2 is 17, since she counted on 2 using her fingers.

#### Counting Back

Keeping 12 in his head, the student counts backwards, "11" as he holds up one finger; says "10" as he holds up a second finger; says "9" as he holds up a third finger. Seeing that he has counted back 3 since he is holding up 3 fingers, the student states that 12 – 3 = 9.

1.0A.6

Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8+6=8+2+4=10+4=14); decomposing a number leading to a ten (e.g., 13-4=13-3-1=10-1=9); using the relationship between addition and subtraction (e.g., knowing that 8+4=12, one knows 12-8=4); and creating equivalent but easier or known sums (e.g., adding 6+7 by creating the known equivalent 6+6+1=12+1=13).

In First Grade, students learn about and use various strategies to solve addition and subtraction problems. When students repeatedly use strategies that make sense to them, they internalize facts and develop fluency for addition and subtraction within 10. When students are able to demonstrate fluency within 10, they are accurate, efficient, and flexible. First Graders then apply similar strategies for solving problems within 20, building the foundation for fluency to 20 in Second Grade.

#### **Developing Fluency for Addition & Subtraction within 10**

to get 15.

Example: Two frogs were sitting on a log. 6 more frogs hopped there. How many frogs are sitting on the log now?

#### Counting- On

I started with 6 frogs and then counted up, Sixxxx.... 7, 8. So there are 8 frogs on the log.

$$6 + 2 = 8$$

#### Internalized Fact

There are 8 frogs on the log. I know this because 6 plus 2 equals 8.

$$6 + 2 = 8$$

#### Add and Subtract within 20

Example: Sam has 8 red marbles and 7 green marbles. How many marbles does Sam have in all?

Making 10 and Decomposing a Number
I know that 8 plus 2 is 10, so I broke up
(decomposed) the 7 up into a 2 and a 5. First I
added 8 and 2 to get 10, and then added the 5

Creating an Easier Problem with Known Sums
I broke up (decomposed) 8 into 7 and 1. I
know that 7 and 7 is 14. I added 1 more to get
15.

Example: There were 14 birds in the tree. 6 flew away. How many birds are in the tree now?

#### Back Down Through Ten

I know that 14 minus 4 is 10. So, I broke the 6 up into a 4 and a 2. 14 minus 4 is 10. Then I took away 2 more to get 8.

Relationship between Addition & Subtraction
I thought, '6 and what makes 14?'. I know that
6 plus 6 is 12 and two more is 14. That's 8
altogether. So, that means that 14 minus 6 is 8.

1.0A.7	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? $6 = 6$ , $7 = 8 - 1$ , $5 + 2$
	= 2 + 5, 4 + 1 = 5 + 2.

In order to determine whether an equation is true or false, First Grade students must first understand the meaning of the equal sign. This is developed as students in Kindergarten and First Grade solve numerous joining and separating situations with mathematical tools, rather than symbols. Once the concepts of joining, separating, and "the same amount/quantity as" are developed concretely, First Graders are ready to connect these experiences to the corresponding symbols (+, -, =). Thus, students learn that the equal sign does not mean "the answer comes next", but that the symbol signifies an equivalent relationship that the left side 'has the same value as' the right side of the equation.

When students understand that an equation needs to "balance", with equal quantities on both sides of the equal sign, they understand various representations of equations, such as:

- an operation on the left side of the equal sign and the answer on the right side (5 + 8 = 13)
- an operation on the right side of the equal sign and the answer on the left side (13 = 5 + 8)
- numbers on both sides of the equal sign (6 = 6)
- operations on both sides of the equal sign (5 + 2 = 4 + 3).

Once students understand the meaning of the equal sign, they are able to determine if an equation is true (9 = 9) or false (9 = 8).

1.0A.8

Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations 8 + ? = 11,  $5 = \_ - 3$ ,  $6 + 6 = \_$ .

First Graders use their understanding of and strategies related to addition and subtraction as described in 1.0A.4 and 1.0A.6 to solve equations with an unknown. Rather than symbols, the unknown symbols are boxes or pictures.

Example: Five cookies were on the table. I ate some cookies. Then there were 3 cookies. How many cookies did I eat?

**Student A:** What goes with 3 to make 5? 3 and 2 is 5. So, 2 cookies were eaten.

**Student B**: Fiiivee, four, three (*holding up 1 finger for each count*). 2 cookies were eaten (*showing 2 fingers*).

**Student C**: We ended with 3 cookies. Threeeee, four, five (*holding up 1 finger for each count*). 2 cookies were eaten (*showing 2 fingers*).

Example: Determine the unknown number that makes the equation true.  $5 - \square = 2$ 

**Student**: 5 minus something is the same amount as 2. Hmmm. 2 and what makes 5? 3! So, 5 minus 3 equals 2.

Now it's true!

	Count to 120, starting at any number less than 120. In this range, read and write numerals and represent a number of objects
1.NBT.1	with a written numeral.

- Count on from a number ending at any number up to 120.
- Recognize and explain patterns with numerals on a hundreds chart.
- Understand that the place of a digit determines its value. For example, students recognize that 24 is different from and less than 42.)
- Explain their thinking with a variety of examples.
- Read and write numerals to 120.

Students extend the range of counting numbers, focusing on the patterns evident in written numerals. This is the foundation for thinking about place value and the meaning of the digits in a numeral. Students are also expected to read and write numerals to 120.

## Math In Focus Lesson Structure

	LESSON STRUCTURE	RESOURCES	COMMENTS
PRE TEST	LESSON STRUCTURE  Chapter Opener Assessing Prior Knowledge  The Pre Test serves as a diagnostic test of readiness of the upcoming chapter		Recall Prior Knowledge (RPK) can take place just before the pre-tests are given and can take 1-2 days to front load prerequisite understanding  Quick Check can be done in concert with the RPK and used to repair student misunderstandings and vocabulary prior to the pre-test; Students write Quick Check answers on a separate sheet of paper  Quick Check and the Pre Test can be done in the same block (See Anecdotal Checklist; Transition
DIRECT ENGAGEMENT	Direct Involvement/Engagement Teach/Learn  Students are directly involved in making sense, themselves, of the concepts – by interacting the tools,	Teacher Edition 5-minute warm up Teach; Anchor Task  Technology Digi Other	The Warm Up activates prior knowledge for each new lesson Student Books are CLOSED; Big Book is used in Gr. K Teacher led; Whole group Students use concrete manipulatives to explore concepts A few select parts of the task are explicitly
DIREC	manipulatives, each other, and the questions  Guided Learning and Practice Guided Learning	Fluency Practice  Teacher Edition Learn	shown, but the majority is addressed through the hands-on, constructivist approach and questioning Teacher facilitates; Students find the solution Students-already in pairs /small, homogenous ability groups; Teacher circulates between
GUIDED LEARNING		Technology Digi  Student Book Guided Learning Pages Hands-on Activity	groups; Teacher, anecdotally, captures student thinking  Small Group w/Teacher circulating among groups Revisit Concrete and Model Drawing; Reteach Teacher spends majority of time with struggling learners; some time with on level, and less time with advanced groups Games and Activities can be done at this time

INDEPENDENT PRACTICE	Independent Practice  A formal formative assessment	Teacher Edition Let's Practice  Student Book Let's Practice  Differentiation Options All: Workbook Extra Support: Reteach On Level: Extra Practice Advanced: Enrichment	Let's Practice determines readiness for Workbook and small group work and is used as formative assessment; Students not ready for the Workbook will use Reteach. The Workbook is continued as Independent Practice.  Manipulatives CAN be used as a communications tool as needed.  Completely Independent On level/advance learners should finish all workbook pages.
RACTICE	Extending the Lesson	Math Journal Problem of the Lesson Interactivities Games	
ADDITIONAL PRACTICE	Lesson Wrap Up	Problem of the Lesson Homework (Workbook , Reteach, or Extra Practice)	Workbook or Extra Practice Homework is only assigned when students fully understand the concepts (as additional practice) Reteach Homework (issued to struggling learners) should be checked the next day
POST TEST	End of Chapter Wrap Up and Post Test	Teacher Edition Chapter Review/Test Put on Your Thinking Cap  Student Workbook Put on Your Thinking Cap  Assessment Book Test Prep	Use Chapter Review/Test as "review" for the End of Chapter Test Prep. Put on your Thinking Cap prepares students for novel questions on the Test Prep; Test Prep is graded/scored. The Chapter Review/Test can be completed  Individually (e.g. for homework) then reviewed in class  As a 'mock test' done in class and doesn't count  As a formal, in class review where teacher walks students through the questions  Test Prep is completely independent; scored/graded Put on Your Thinking Cap (green border) serve as a capstone problem and are done just before the Test Prep and should be treated as Direct Engagement. By February,

### Math Background:

- Countable items are used to develop the association between the physical representation of the number, the number symbol and the number- word.
- Students need to understand the sequential order of their counting numbers and their relative magnitudes, such as one more and one less than the item before.
- When children understand the concept of number bonds, it will be easier when they do addition and regrouping at a later stage.
- Children are first taught to identify the parts and whole in a set and then led to separate a set of objects into two parts.
- Addition is one of the four basic operations that form the foundation or arithmetic and is an essential part of the computation work in elementary school.
- Addition stories are constructed about consumable items in pictures to help children write addition sentences.
- The subtraction strategy that builds and reinforces this concept most effectively involves the use of number bonds, which relate parts with the whole, and relate addition with subtraction.

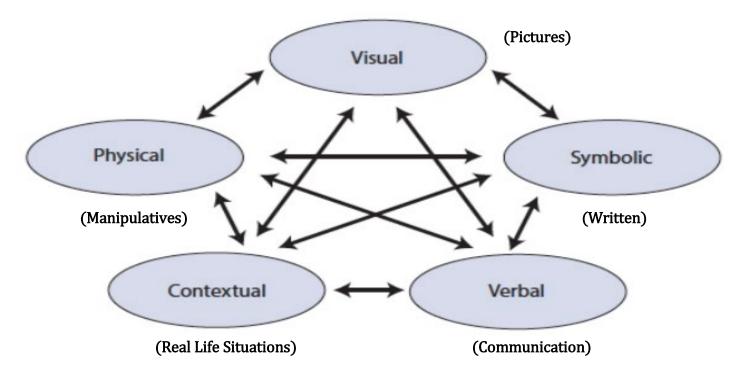
### **Misconceptions:**

- Children may not understand that activities call for counting the number of objects in different groups.
- Children may confuse the terms greater than and less than and struggle in comparing the magnitude of two numbers from the numerals alone.
- Children may confuse the parts of a number bond and record numbers in the wrong places.
- Some children may lose track of the numbers when they count on. First, have them record the number they are counting on from.
- Some children may correctly identify the numbers to use in the story problems, but incorrectly add. Review how to use objects or count on to add.
- Some children may confuse the two subtraction sentences for a given number bond. If they write the sentences 5-4=1, have them reread the question that was asked. Be sure they understand that the answer to the question should be the answer to the subtraction problem.

	PARCC Assessment Evidence/Clarification Statements			
ccss	Evidence Statement	Clarification	Math Practices	
1.OA.A.1	Use addition and subtraction within 20 to solve word problems involving situations of adding to, taking from, putting together, taking apart and comparing, with unknown in all positions, e.g., by using objects, drawings, and equations with a symbol for the unknown number to represent the problem.	i) Tasks should include all problem situations and all of their subtypes and language variants. Mastery is expected in "Add To" and "Take From" - Result and Change Unknown Problems, "Put Together/Take Apart" Problems, "Compare" – Difference Unknown, Bigger Unknown (more version) and Smaller Unknown (fewer version) Problems (for more information see CCSS Table 1 and OA Progression, p. 9.) ii) Interviews (individual or small group) are used to assess mastery of different problem types.	MP 1, 4	
1.OA.B.3	Apply properties of operations as strategies to add and subtract. Examples: if 8+3 = 11 is known, then 3+8 = 11 is also known (Commutative property of addition). To add 2+6+4, the second two numbers can be added to make a ten, so 2+6+4 = 2+10 = 12 (Associative property of addition).	i)Tasks should not expect students to know the names of the properties. ii) Interviews (individual or small group) should target students' application of properties of operations to add and subtract.	MP 7,8	
1.OA.D.7	Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8-1, 5+2=2+5, 4+1=5+2.	i) Interviews (individual or small group) should target students' understanding of the equal sign.	MP 7,8	
1.OA.D.8	Determine the unknown whole number in an addition or subtraction equation relating three whole numbers. For example, determine the unknown number that makes the equation true in each of the equations	i) Interviews (individual or small group) should target students' thinking strategies for determining the unknown in an addition or subtraction equation relating 3 whole numbers. Thinking strategies expected in Grade 1 (Level 2 and 3)	MP 7,8	

, ,	are defined in 1.OA.6 and in OA Progression	

#### **Use and Connection of Mathematical Representations**



The Lesh Translation Model

Each oval in the model corresponds to one way to represent a mathematical idea.

**Visual:** When children draw pictures, the teacher can learn more about what they understand about a particular mathematical idea and can use the different pictures that children create to provoke a discussion about mathematical ideas. Constructing their own pictures can be a powerful learning experience for children because they must consider several aspects of mathematical ideas that are often assumed when pictures are pre-drawn for students.

**Physical**: The manipulatives representation refers to the unifix cubes, base-ten blocks, fraction circles, and the like, that a child might use to solve a problem. Because children can physically manipulate these objects, when used appropriately, they provide opportunities to compare relative sizes of objects, to identify patterns, as well as to put together representations of numbers in multiple ways.

**Verbal**: Traditionally, teachers often used the spoken language of mathematics but rarely gave students opportunities to grapple with it. Yet, when students do have opportunities to express their mathematical reasoning aloud, they may be able to make explicit some knowledge that was previously implicit for them.

**Symbolic**: Written symbols refer to both the mathematical symbols and the written words that are associated with them. For students, written symbols tend to be more abstract than the other representations. I tend to introduce symbols after students have had opportunities to make connections among the other representations, so that the students have multiple ways to connect the symbols to mathematical ideas, thus increasing the likelihood that the symbols will be comprehensible to students.

**Contextual:** A relevant situation can be any context that involves appropriate mathematical ideas and holds interest for children; it is often, but not necessarily, connected to a real-life situation.

#### The Lesh Translation Model: Importance of Connections

As important as the ovals are in this model, another feature of the model is even more important than the representations themselves: The arrows! The arrows are important because they represent the connections students make between the representations. When students make these connections, they may be better able to access information about a mathematical idea, because they have multiple ways to represent it and, thus, many points of access.

Individuals enhance or modify their knowledge by building on what they already know, so the greater the number of representations with which students have opportunities to engage, the more likely the teacher is to tap into a student's prior knowledge. This "tapping in" can then be used to connect students' experiences to those representations that are more abstract in nature (such as written symbols). Not all students have the same set of prior experiences and knowledge. Teachers can introduce multiple representations in a meaningful way so that students' opportunities to grapple with mathematical ideas are greater than if their teachers used only one or two representations.

#### Concrete Pictorial Abstract (CPA) Instructional Approach

The CPA approach suggests that there are three steps necessary for pupils to develop understanding of a mathematical concept.

**Concrete:** "Doing Stage": Physical manipulation of objects to solve math problems.

**Pictorial:** "Seeing Stage": Use of imaged to represent objects when solving math problems.

**Abstract:** "Symbolic Stage": Use of only numbers and symbols to solve math problems.

CPA is a gradual systematic approach. Each stage builds on to the previous stage. Reinforcement of concepts are achieved by going back and forth between these representations and making connections between stages. Students will benefit from seeing parallel samples of each stage and how they transition from one to another.

#### Read, Draw, Write Process

**READ** the problem. Read it over and over.... And then read it again.

**DRAW** a picture that represents the information given. During this step students ask themselves: Can I draw something from this information? What can I draw? What is the best model to show the information? What conclusions can I make from the drawing?

**WRITE** your conclusions based on the drawings. This can be in the form of a number sentence, an equation, or a statement.

Students are able to draw a model of what they are reading to help them understand the problem. Drawing a model helps students see which operation or operations are needed, what patterns might arise, and which models work and do not work. Students must dive deeper into the problem by drawing models and determining which models are appropriate for the situation.

While students are employing the RDW process they are using several Standards for Mathematical Practice and in some cases, all of them.

#### Mathematical Discourse and Strategic Questioning

Discourse involves asking strategic questions that elicit from students both how a problem was solved and why a particular method was chosen. Students learn to critique their own and others' ideas and seek out efficient mathematical solutions.

While classroom discussions are nothing new, the theory behind classroom discourse stems from constructivist views of learning where knowledge is created internally through interaction with the environment. It also fits in with socio-cultural views on learning where students working together are able to reach new understandings that could not be achieved if they were working alone.

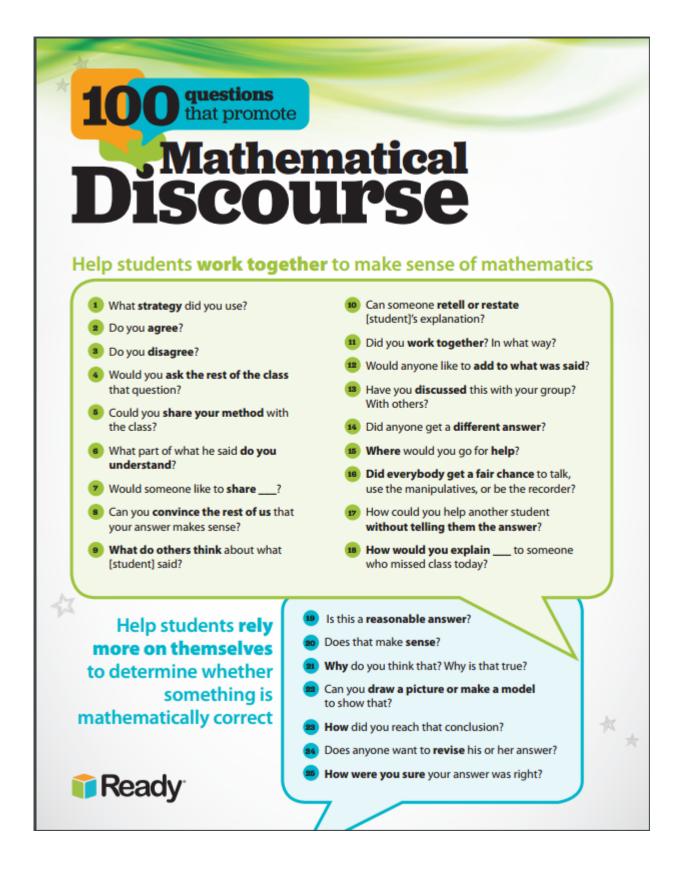
Underlying the use of discourse in the mathematics classroom is the idea that mathematics is primarily about reasoning not memorization. Mathematics is not about remembering and applying a set of procedures but about developing understanding and explaining the processes used to arrive at solutions.

Asking better questions can open new doors for students, promoting mathematical thinking and classroom discourse. Can the questions you're asking in the mathematics classroom be answered with a simple "yes" or "no," or do they invite students to deepen their understanding?



Albert Einstein

To help you encourage deeper discussions, here are 100 questions to incorporate into your instruction by Dr. Gladis Kersaint, mathematics expert and advisor for Ready Mathematics.



### Help students learn to reason mathematically

- How did you begin to think about this problem?
- What is another way you could solve this problem?
- How could you prove \_\_\_\_\_?
- Can you explain how your answer is different from or the same as [student]'s answer?
- Let's break the problem into parts. What would the parts be?
- Can you explain this part more specifically?
- Does that always work?
- Can you think of a case where that wouldn't work?
- 34 How did you organize your information? Your thinking?

Help students with problem comprehension

Help students evaluate their own processes and engage in productive peer interaction

- What do you need to do next?
- What have you accomplished?
- What are your strengths and weaknesses?
- Was your group participation appropriate and helpful?
  - What is this problem about? What can you tell me about it?
  - Do you need to define or set limits for the problem?
  - 41 How would you interpret that?
  - Could you reword that in simpler terms?
  - Is there something that can be eliminated or that is missing?
  - Could you explain what the problem is asking?
  - What assumptions do you have to make?
  - What do you know about this part?
  - Which words were most important? Why?



100 Questions That Promote Mathematical Discourse 2



#### Help students learn to conjecture, invent, and solve problems

- What would happen if \_\_\_\_?
- Do you see a pattern?
- What are some possibilities here?
- Where could you find the information you need?
- How would you check your steps or your answer?
- What did not work?
- 60 How is your solution method the same as or different from [student]'s method?
- Other than retracing your steps, how can you determine if your answers are appropriate?
- 66 How did you organize the information? Do you have a record?
- How could you solve this using tables, lists, pictures, diagrams, etc.?
- What have you tried? What steps did you take?
- 69 How would it look if you used this model or these materials?

- How would you draw a diagram or make a sketch to solve the problem?
- 61 Is there another possible answer? If so, explain.
- Is there another way to solve the problem?
- Is there another model you could use to solve the problem?
- Is there anything you've overlooked?
- How did you think about the problem?
- 66 What was your estimate or prediction?
- How confident are you in your answer?
- What else would you like to know?
- What do you think comes next?
- Is the solution reasonable, considering the context?
- Did you have a system? Explain it.
- Did you have a strategy? Explain it.
- Did you have a design? Explain it.





100 Questions That Promote Mathematical Discourse 3

# Help students learn to connect mathematics, its ideas, and its application

- What is the **relationship** between \_\_\_ and \_\_\_?
- Have we ever solved a problem like this before?
- What uses of mathematics did you find in the **newspaper** last night?
- What is the same?
- What is different?
- Did you use skills or build on concepts that were not necessarily mathematical?
- 80 Which skills or concepts did you use?
- What ideas have we explored before that were useful in solving this problem?

- 82 Is there a pattern?
- Where else would this strategy be useful?
- How does this relate to \_\_\_\_?
- Is there a general rule?
- Is there a real-life situation where this could be used?
- How would your method work with other problems?
- What other problem does this seem to lead to?
  - Have you tried making a guess?
  - 90 What else have you tried?
  - Would another method work as well or better?
  - Is there another way to draw, explain, or say that?
  - Give me another related problem. Is there an easier problem?
  - How would you explain what you know right now?

# Help students persevere

- What was one thing you learned (or two, or more)?
- Did you notice any patterns? If so, describe them.
- What mathematics topics were used in this investigation?
- What were the mathematical ideas in this problem?
- What is mathematically different about these two situations?
- What are the variables in this problem? What stays constant?

Help students focus on the mathematics from activities

Ready

100 Questions That Promote Mathematical Discourse 4

# **Conceptual Understanding**

Students demonstrate conceptual understanding in mathematics when they provide evidence that they can:

- recognize, label, and generate examples of concepts;
- use and interrelate models, diagrams, manipulatives, and varied representations of concepts;
- identify and apply principles; know and apply facts and definitions;
- compare, contrast, and integrate related concepts and principles; and
- recognize, interpret, and apply the signs, symbols, and terms used to represent concepts.

Conceptual understanding reflects a student's ability to reason in settings involving the careful application of concept definitions, relations, or representations of either.

## **Procedural Fluency**

Procedural fluency is the ability to:

- apply procedures accurately, efficiently, and flexibly;
- to transfer procedures to different problems and contexts;
- to build or modify procedures from other procedures; and
- to recognize when one strategy or procedure is more appropriate to apply than another.

Procedural fluency is more than memorizing facts or procedures, and it is more than understanding and being able to use one procedure for a given situation. Procedural fluency builds on a foundation of conceptual understanding, strategic reasoning, and problem solving (NGA Center & CCSSO, 2010; NCTM, 2000, 2014). Research suggests that once students have memorized and practiced procedures that they do not understand, they have less motivation to understand their meaning or the reasoning behind them (Hiebert, 1999). Therefore, the development of students' conceptual understanding of procedures should precede and coincide with instruction on procedures.

## Math Fact Fluency: Automaticity

Students who possess math fact fluency can recall math facts with automaticity. Automaticity is the ability to do things without occupying the mind with the low-level details required, allowing it to become an automatic response pattern or habit. It is usually the result of learning, repetition, and practice.

#### K-2 Math Fact Fluency Expectation

**K.OA.5** Add and Subtract within 5.

**1.0A.6** Add and Subtract within 10.

**2.0A.2** Add and Subtract within 20.

# Math Fact Fluency: Fluent Use of Mathematical Strategies

First and second grade students are expected to solve addition and subtraction facts using a variety of strategies fluently.

**1.0A.6** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10.

Use strategies such as:

- counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14);
- decomposing a number leading to a ten (e.g., 13 4 = 13 3 1 = 10 1 = 9);
- using the relationship between addition and subtraction; and
- creating equivalent but easier or known sums.

**2.NBT.7** Add and subtract within 1000, using concrete models or drawings and strategies based on:

- o place value,
- o properties of operations, and/or
- o the relationship between addition and subtraction;

# **Evidence of Student Thinking**

Effective classroom instruction and more importantly, improving student performance, can be accomplished when educators know how to elicit evidence of students' understanding on a daily basis. Informal and formal methods of collecting evidence of student understanding enable educators to make positive instructional changes. An educators' ability to understand the processes that students use helps them to adapt instruction allowing for student exposure to a multitude of instructional approaches, resulting in higher achievement. By highlighting student thinking and misconceptions, and eliciting information from more students, all teachers can collect more representative evidence and can therefore better plan instruction based on the current understanding of the entire class.

#### **Mathematical Proficiency**

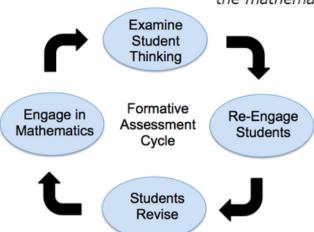
To be mathematically proficient, a student must have:

- Conceptual understanding: comprehension of mathematical concepts, operations, and relations;
- <u>Procedural fluency</u>: skill in carrying out procedures flexibly, accurately, efficiently, and appropriately;
- <u>Strategic competence</u>: ability to formulate, represent, and solve mathematical problems;
- Adaptive reasoning: capacity for logical thought, reflection, explanation, and justification;
- <u>Productive disposition</u>: habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

#### Evidence should:

- Provide a window in student thinking;
- Help teachers to determine the extent to which students are reaching the math learning goals; and
- Be used to make instructional decisions during the lesson and to prepare for subsequent lessons.

Formative assessment is an essentially interactive process, in which the teacher can find out whether what has been taught has been learned, and if not, to do something about it. Day-to-day formative assessment is one of the most powerful ways of improving learning in the mathematics classroom.



(Wiliam 2007, pp. 1054; 1091)

Unit 1 Assessment / Authentic Assessment Framework			
Assessment	ccss	Estimated Time	Format
Diagnostic Assessment (IREADY)		1-2 blocks	Individual
Chapter 1			
Optional Pre Test1	K.CC.4,6	½ block	Individual
Optional Chapter 1 Test/	1.NBT.1	1 block	Individual
Authentic Assessment #1  Student Page: https://www- k6.thinkcentral.com/content/hsp/math/mathinfocus/na/g r1/performance_tasks_9780544618237_/pdf/GR1_CH1_P T_S.pdf Teacher Pages: https://www- k6.thinkcentral.com/content/hsp/math/mathinfocus/na/g r1/performance_tasks_9780544618237_/pdf/GR1_CH1_P T_T.pdf	1.NBT.1	½ block	Individual
Chapter 2			
Optional Pre Test 2	1.OA.3,6	½ block	Individual
Optional Chapter 2 Test/ Performance Task	1.OA.3,6	1 block	Individual
Chapter 3			
Optional Pre-Test 3	1.OA.1,3,5-8	½ block	Individual
Optional Chapter 3 Test	1.OA.1,3,5-8	1 block	Individual
Authentic Assessment #2 (Performance Task Chapter 3) Student Pages: <a href="https://www-k6.thinkcentral.com/content/hsp/math/mathinfocus/na/gr1/performance_tasks_9780544618237_/pdf/GR1_CH3_PT_S.pdf">https://www-k6.thinkcentral.com/content/hsp/math/mathinfocus/na/gr1/performance_tasks_9780544618237_/pdf/GR1_CH3_PT_T.pdf</a>	1.OA.1,3,5-8	½ block	Pairs
Chapter 4			
Optional Pre-Test 4	1.OA.1,4-8	½ block	Individual
Optional Chapter 4 Test/ Performance Task	1.OA.1,4-8	1 block	Individual

Authentic Assessment #3	1.OA.7	½ Block	Individual
Eureka Math Module 1: Sums and Differences to 10 (Topic E,F,G,HI)			
Optional End of Module Assessment	1.OA.1-8	1block	Individual
Grade 1 Interim Assessment (Interview Style)	1.OA.1-8	½ block	Individual or Small Group with Teacher

	PLD	Genesis Conversion
Rubric Scoring	PLD 5	100
	PLD 4	89
	PLD 3	79
	PLD 2	69
	PLD 1	59

# **Connections to the Mathematical Practices**

# Student Friendly Connections to the Mathematical Practices

- 1. I can solve problems without giving up.
- 2. I can think about numbers in many ways.
- 3. I can explain my thinking and try to understand others.
- 4. I can show my work in many ways.
- 5. I can use math tools and tell why I choose them.
- 6. I can work carefully and check my work.
- 7. I can use what I know to solve new problems.
- 8. I can discover and use short cuts.

The **Standards for Mathematical Practice** describe varieties of expertise that mathematics educators at all levels should seek to develop in their students.

#### Make sense of problems and persevere in solving them

Mathematically proficient students in First Grade continue to develop the ability to focus attention, test hypotheses, take reasonable risks, remain flexible, try alternatives, exhibit self-regulation, and persevere (Copley, 2010). As the teacher uses thoughtful questioning and provides opportunities for students to share thinking, First Grade students become conscious of what they know and how they solve problems. They make sense of task-type problems, find an entry point or a way to begin the task, and are willing to try other approaches when solving the task. They ask themselves, "Does this make sense?" First Grade students' conceptual understanding builds from their experiences in Kindergarten as they continue to rely on concrete manipulatives and pictorial representations to solve a problem, eventually becoming fluent and flexible with mental math as a result of these experiences..

### Reason abstractly and quantitatively

Mathematically proficient students in First Grade recognize that a number represents a specific quantity. They use numbers and symbols to represent a problem, explain thinking, and justify a response. For example, when solving the problem: "There are 60 children on the playground. Some children line up. There are 20 children still on the playground. How many children lined up?" first grade students may write 20 + 40 = 60 to indicate a Think-Addition strategy. Other students may illustrate a counting-on by tens strategy by writing 20 + 10 + 10 + 10 + 10 = 60. The numbers and equations written illustrate the students' thinking and the strategies used, rather than how to simply compute, and how the story is decontextualized as it is represented abstractly with symbols.

## Construct viable arguments and critique the reasoning of others

Mathematically proficient students in First Grade continue to develop their ability to clearly express, explain, organize and consolidate their math thinking using both verbal and written representations. Their understanding of grade appropriate vocabulary helps them to construct viable arguments about mathematics. For example, when justifying why a particular shape isn't a square, a first grade student may hold up a picture of a rectangle, pointing to the various parts, and reason, "It can't be a square because, even though it has 4 sides and 4 angles, the sides aren't all the same size." In a classroom where risk-taking and varying perspectives are encouraged, mathematically proficient students are willing and eager to share their ideas with others, consider other ideas proposed by classmates, and question ideas that don't seem to make sense.

#### Model with mathematics

Mathematically proficient students in First Grade model real-life mathematical situations with a number sentence or an equation, and check to make sure that their equation accurately matches the problem context. They also use tools, such as tables, to help collect information, analyze results, make conclusions, and review their conclusions to see if the results make sense and revising as needed.

4

#### Use appropriate tools strategically

Mathematically proficient students in First Grade have access to a variety of concrete (e.g. 3-dimensional solids, ten frames, number balances, number lines) and technological tools (e.g., virtual manipulatives, calculators, interactive websites) and use them to investigate mathematical concepts. They select tools that help them solve and/or illustrate solutions to a problem. They recognize that multiple tools can be used for the same problem- depending on the strategy used. For example, a child who is in the counting stage may choose connecting cubes to solve a problem. While, a student who understands parts of number, may solve the same problem using ten-frames to decompose numbers rather than using individual connecting cubes. As the teacher provides numerous opportunities for students to use educational materials, first grade students' conceptual understanding and higher order thinking skills are developed

#### Attend to precision

Mathematically proficient students in First Grade attend to precision in their communication, calculations, and measurements. They are able to describe their actions and strategies clearly, using grade-level appropriate vocabulary accurately. Their explanations and reasoning regarding their process of finding a solution becomes more precise. In varying types of mathematical tasks, first grade students pay attention to details as they work. For example, as students' ability to attend to position and direction develops, they begin to notice reversals of numerals and self-correct when appropriate. When measuring an object, students check to make sure that there are not any gaps or overlaps as they carefully place each unit end to end to measure the object (iterating length units). Mathematically proficient first grade students understand the symbols they use (=, >, 3, a proficient student who is able to attend to precision states, "Four is more than 3" rather than "The alligator eats the four. It's bigger."

#### Look for and make use of structure

Mathematically proficient students in First Grade carefully look for patterns and structures in the number system and other areas of mathematics. For example, while solving addition problems using a number balance, students recognize that regardless whether you put the 7 on a peg first and then the 4, or the 4 on first and then the 7, they both equal 11 (commutative property). When decomposing two-digit numbers, students realize that the number of tens they have constructed 'happens' to coincide with the digit in the tens place. When exploring geometric properties, first graders recognize that certain attributes are critical (number of sides, angles), while other properties are not (size, color, orientation)

### Look for and express regularity in repeated reasoning

Mathematically proficient students in First Grade begin to look for regularity in problem structures when solving mathematical tasks. For example, when adding three one-digit numbers and by making tens or using doubles, students engage in future tasks looking for opportunities to employ those same strategies. Thus, when solving 8+7+2, a student may say, "I know that 8 and 2 equal 10 and then I add 7 more. That makes 17. It helps to see if I can make a 10 out of 2 numbers when I start." Further, students use repeated reasoning while solving a task with multiple correct answers. For example, in the task "There are 12 crayons in the box. Some are red and some are blue. How many of each could there be?" First Grade students realize that the 12 crayons could include 6 of each color (6+6 = 12), 7 of one color and 5 of another (7+5 = 12), etc. In essence, students repeatedly find numbers that add up to 12.

# **Effective Mathematics Teaching Practices**

**Establish mathematics goals to focus learning.** Effective teaching of mathematics establishes clear goals for the mathematics that students are learning, situates goals within learning progressions, and uses the goals to guide instructional decisions.

Implement tasks that promote reasoning and problem solving. Effective teaching of mathematics engages students in solving and discussing tasks that promote mathematical reasoning and problem solving and allow multiple entry points and varied solution strategies.

Use and connect mathematical representations. Effective teaching of mathematics engages students in making connections among mathematical representations to deepen understanding of mathematics concepts and procedures and as tools for problem solving.

Facilitate meaningful mathematical discourse. Effective teaching of mathematics facilitates discourse among students to build shared understanding of mathematical ideas by analyzing and comparing student approaches and arguments.

**Pose purposeful questions**. Effective teaching of mathematics uses purposeful questions to assess and advance students' reasoning and sense making about important mathematical ideas and relationships.

**Build procedural fluency from conceptual understanding**. Effective teaching of mathematics builds fluency with procedures on a foundation of conceptual understanding so that students, over time, become skillful in using procedures flexibly as they solve contextual and mathematical problems.

**Support productive struggle in learning mathematics**. Effective teaching of mathematics consistently provides students, individually and collectively, with opportunities and supports to engage in productive struggle as they grapple with mathematical ideas and relationships.

Elicit and use evidence of student thinking. Effective teaching of mathematics uses evidence of student thinking to assess progress toward mathematical understanding and to adjust instruction continually in ways that support and extend learning.

5 Practices for Orchestrating Productive Mathematics Discussions			
Practice	Description/ Questions		
1. Anticipating	What strategies are students likely to use to approach or solve a challenging high-level mathematical task?		
	How do you respond to the work that students are likely to produce?		
	Which strategies from student work will be most useful in addressing the mathematical goals?		
2. Monitoring	Paying attention to what and how students are thinking during the lesson.		
	Students working in pairs or groups		
	Listening to and making note of what students are discussing and the strategies they are using		
	Asking students questions that will help them stay on track or help them think more deeply about the task. (Promote productive struggle)		
3. Selecting	This is the process of deciding the <i>what</i> and the <i>who</i> to focus on during the discussion.		
4. Sequencing	What order will the solutions be shared with the class?		
5. Connecting	Asking the questions that will make the mathematics explicit and understandable.		
	Focus must be on mathematical meaning and relationships; making links between mathematical ideas and representations.		

# K-4 Ideal Math Block

# Daily Math Routines: Whole Group

EX: Number Strings, Sprints, Counting, Whiteboard Exchange

# Anchor Task/ Application Problem:

Provides HANDS-ON work to allow children to ACT OUT/ EXPLORE/ ENGAGE
ACTIVELY with the new MATH IDEA

60 min.

# Concept Development: Individual/partner/whole

Instruction & Strategic Problem Set Questions Independent Practice

Student Debrief: Whole Group

Exit Ticket: Independent

# CENTERS/ LEARNING STATIONS:

Pairs / Small Group/ Individual

**DIFFERENTIATED** activities designed to **RETEACH**, **REMEDIATE**, **ENRICH** student's understanding of concepts.

30 min.

M: Meet with the teacher A: Application/ Problem Solving

T: Technology H: Hands on Activities

# First Grade PLD Rubric

Go	+ I+	Not There Yet		
Evidence shows that the		Student shows evidence	of a major misunderstand	ding incorrect concents
the target concept or big		or procedure, or a failure		anig, incorrect concepts
PLD Level 5: 100%	PLD Level 4: 89%	PLD Level 3: 79%	PLD Level 2: 69%	PLD Level 1: 59%
Distinguished	Strong Command	Moderate Command	Partial Command	Little Command
command	burong dominana	Trougrate communa		Zituo communa
Student work shows	Student work shows	Student work shows	Student work shows	Student work shows
distinguished levels of	strong levels of	moderate levels of	partial understanding	little understanding of
understanding of the	understanding of the	understanding of the	of the mathematics.	the mathematics.
mathematics.	mathematics.	mathematics.		
			Student constructs	Student attempts to
Student <b>constructs</b>	Student <b>constructs</b>	Student <b>constructs</b> and	and <b>communicates</b> an	constructs and
and <b>communicates</b> a	and <b>communicates</b> a	communicates a	incomplete response	communicates a
complete response	complete response	complete response	based on student's	response using the:
based on	based on	based on	attempts of	• Tools:
explanations/reasonin	explanations/reasoni	explanations/reasonin	explanations/	<ul> <li>Manipula</li> </ul>
g using the:	ng using the:	g using the:	reasoning using the:	tives
• Tools:	• Tools:	• Tools:	• Tools:	o Five
o Manipula	o Manipula	o Manipulat	o Manipula	Frame
tives	tives	ives	tives	o Ten Frame
o Five Frame	○ Five Frame	o Five Frame	o Five Frame	N. 1
m			_	o Number Line
o Ten Frame	o Ten Frame	o Ten Frame	o Ten Frame	o Part-Part-
o Number	o Number	o Number	o Number	Whole
Line	Line	Line	Line	Model
o Part-Part-	o Part-	o Part-Part-	o Part-Part-	Strategies:
Whole	Part-	Whole	Whole	o Drawings
Model	Whole	Model	Model	<ul> <li>Counting</li> </ul>
Strategies:	Model	Strategies:	Strategies:	All
o Drawings	Strategies:	<ul> <li>Drawings</li> </ul>	<ul> <li>Drawings</li> </ul>	<ul> <li>Count</li> </ul>
<ul> <li>Counting</li> </ul>	<ul> <li>Drawings</li> </ul>	<ul> <li>Counting</li> </ul>	<ul> <li>Counting</li> </ul>	On/Back
All	<ul> <li>Counting</li> </ul>	All	All	<ul><li>Skip</li></ul>
o Count	All	o Count	o Count	Counting
On/Back	o Count	On/Back	On/Back	o Making
o Skip	On/Back	o Skip	o Skip	Ten
Counting	o Skip	Counting	Counting	o Decompo
o Making	Counting	o Making	o Making	sing
Ten	o Making Ten	Ten ○ Decompos	Ten	Number
o Decompo	-	•	<ul><li>Decompo sing</li></ul>	Precise use of math vocabulary
sing Number	o Decompo sing	ing Number	Number	mani vocabulary
Precise use of	Number	Precise use of	Precise use of	Response includes
math vocabulary	Precise use of	math vocabulary	math vocabulary	limited evidence of the
Response includes an	math vocabulary	macii vocabalai y	main vocabalary	progression of
efficient and logical		Response includes a	Response includes an	mathematical
progression of	Response includes a	logical but incomplete	incomplete or illogical	reasoning and
mathematical	logical progression of			understanding.
reasoning and	mathematical	mathematical	mathematical	
understanding.	reasoning and	reasoning and	reasoning and	
	understanding.	understanding. understanding.		
		Contains <b>minor errors</b> .		
5 points	4 points	3 points	2 points	1 point

## True or False

Look at each equation. Tell if the equation is true or false by circling the word. Explain your reasoning with pictures, numbers, or words.

$$4 + 1 = 5 + 2$$

True

False

$$3 + 8 = 8 + 3$$

True

False

True

False

$$8 = 7 - 1$$

True

False

**1.OA.7**: Understand the meaning of the equal sign, and determine if equations involving addition and subtraction are true or false. For example, which of the following equations are true and which are false? 6 = 6, 7 = 8 - 1, 5 + 2 = 2 + 5, 4 + 1 = 5 + 2.

Mathematical Practices:

Individual

Level 5: Distinguished	Level 4: Strong	Level 3: Moderate	Level 2: Partial	Level 1: No
Command	Command	Command	Command	Command
Student correctly answers and clearly constructs and communicates a complete response based on explanations/reasoning using:  • Concepts of equality • Strategies based the relationship between addition and subtraction  Response includes an efficient and logical progression of steps.  Strategy and execution meet the content, process, and qualitative demands of the task or concept. Student can communicate ideas.	Student correctly answers and clearly constructs and communicates a complete response with one minor calculation error based on explanations/reasoning using:  • Concepts of equality • Strategies based the relationship between addition and subtraction  Response includes a logical progression of steps  May have minor errors that do not impact the mathematics.	Student answers,  clearly constructs, and communicates a complete response with minor calculation errors based on explanations/reasoning using: • Concepts of equality • Strategies based the relationship between addition and subtraction  Response includes a logical but incomplete progression of steps. Minor calculation errors.	The task is attempted and some mathematical effort is made. There may be fragments of accomplishment but little success.  • Concepts of equality • Strategies based the relationship between addition and subtraction  Response includes an incomplete or Illogical progression of steps.	The student shows no work or justification.  Further teaching is required.

#### Resources

Think Central: <a href="https://www-k6.thinkcentral.com/ePC/start.do">https://www-k6.thinkcentral.com/ePC/start.do</a>

#### **Engage NY**

http://www.engageny.org/video-library?f[0]=im\_field\_subject%3A19

#### **Common Core Tools**

http://commoncoretools.me/

http://www.ccsstoolbox.com/

http://www.achievethecore.org/steal-these-tools

#### Achieve the Core

http://achievethecore.org/dashboard/300/search/6/1/0/1/2/3/4/5/6/7/8/9/10/11/12

#### Manipulatives

http://nlvm.usu.edu/en/nav/vlibrary.html

http://www.explorelearning.com/index.cfm?method=cResource.dspBrowseCorrelations&v=s&id=USA-000 http://www.thinkingblocks.com/

Illustrative Math Project : <a href="http://illustrativemathematics.org/standards/k8">http://illustrativemathematics.org/standards/k8</a>

Inside Mathematics: http://www.insidemathematics.org/index.php/tools-for-teachers

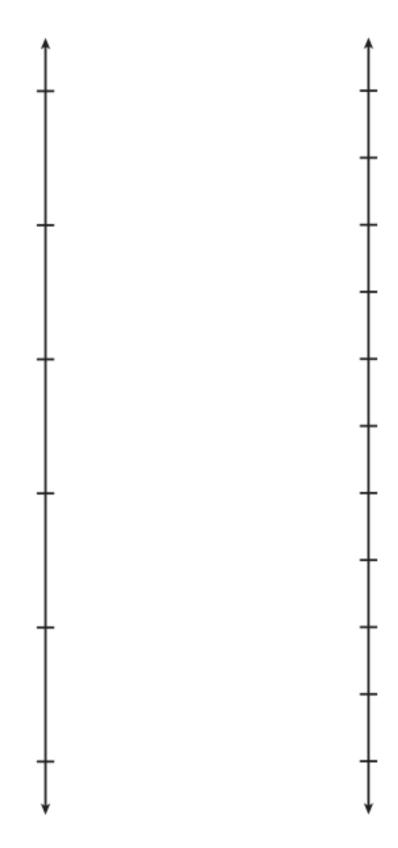
Sample Balance Math Tasks: <a href="http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/">http://www.nottingham.ac.uk/~ttzedweb/MARS/tasks/</a>

Georgia Department of Education: <a href="https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx">https://www.georgiastandards.org/Common-Core/Pages/Math-K-5.aspx</a> Gates Foundations Tasks: <a href="https://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf">https://www.gatesfoundation.org/college-ready-education/Documents/supporting-instruction-cards-math.pdf</a>

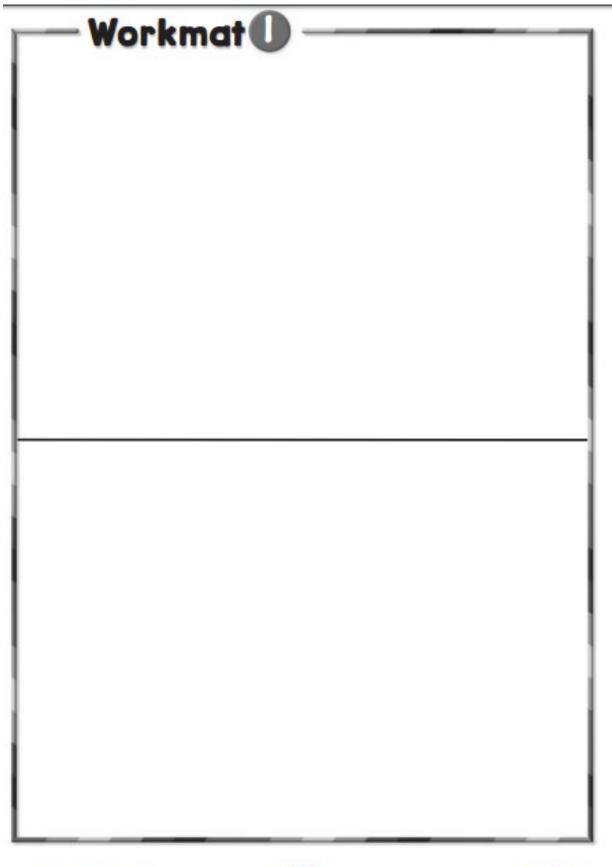
Minnesota STEM Teachers' Center: <a href="http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships">http://www.scimathmn.org/stemtc/frameworks/721-proportional-relationships</a>

Singapore Math Tests K-12: <a href="http://www.misskoh.com">http://www.misskoh.com</a>

Mobymax.com: <a href="http://www.mobymax.com">http://www.mobymax.com</a>



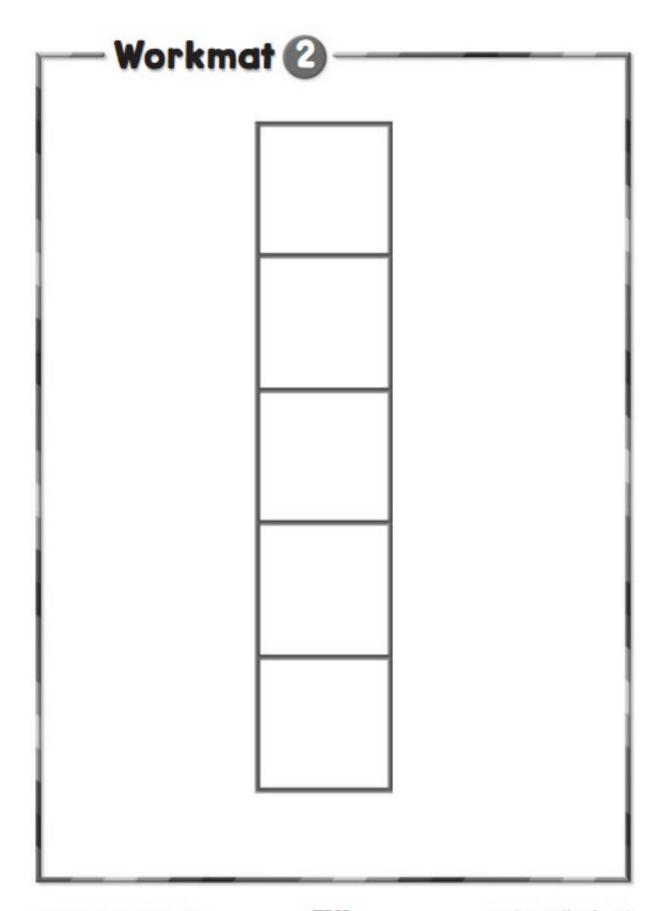
Resources Publishing Company TR61 Number Lines



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TR54

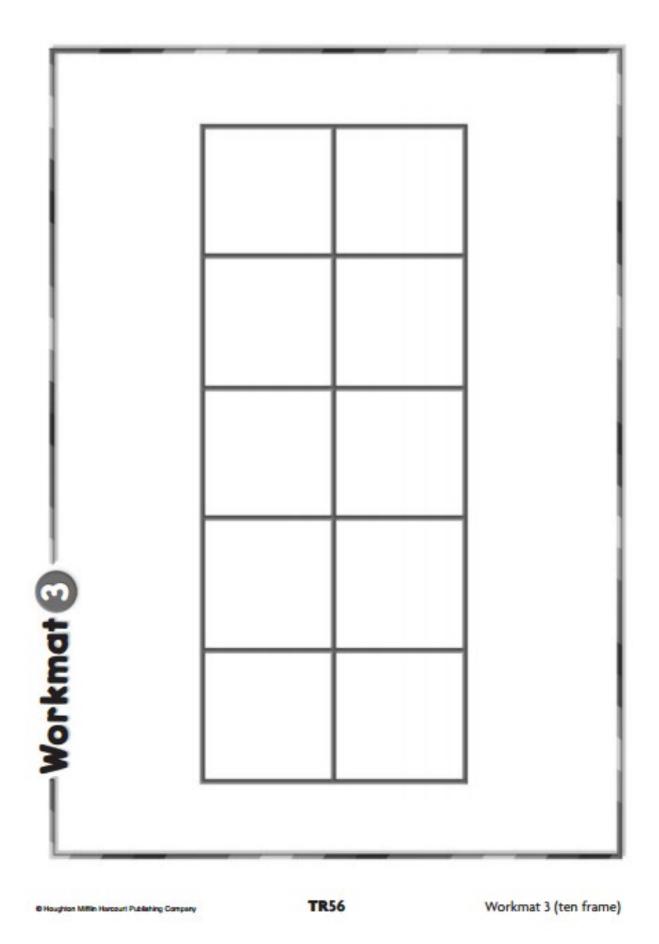
Workmat 1

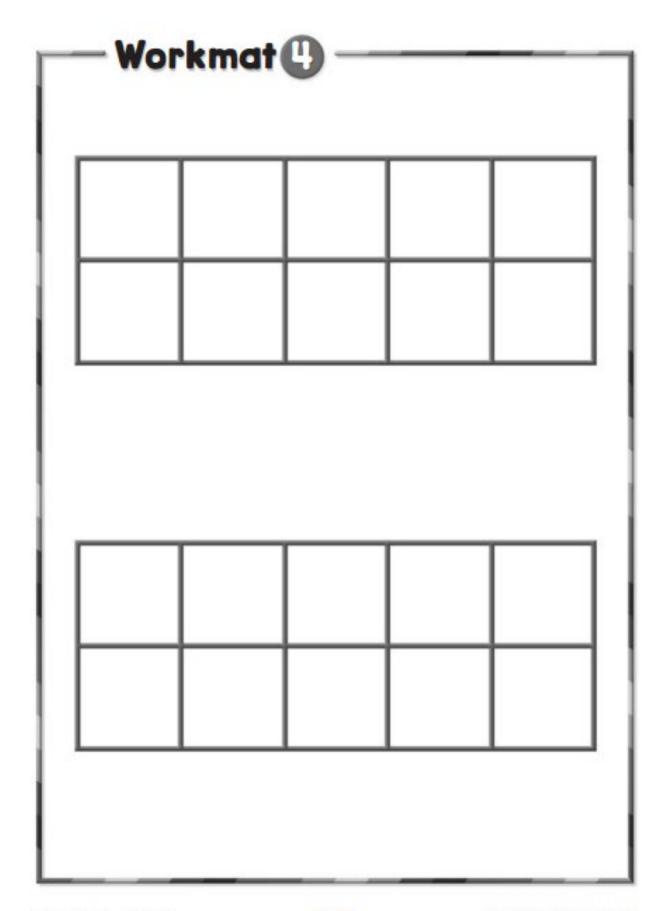


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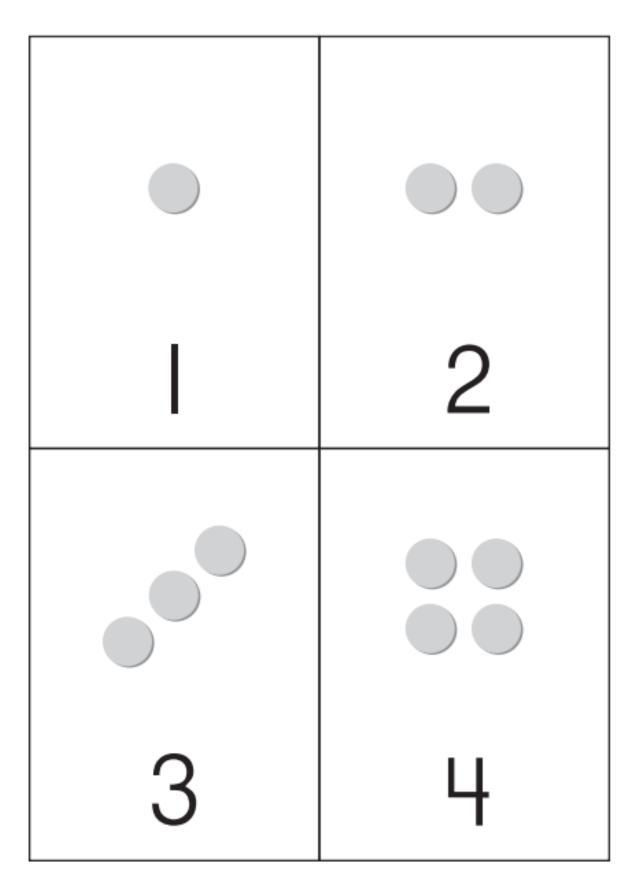
TR55

Workmat 2 (five frame)





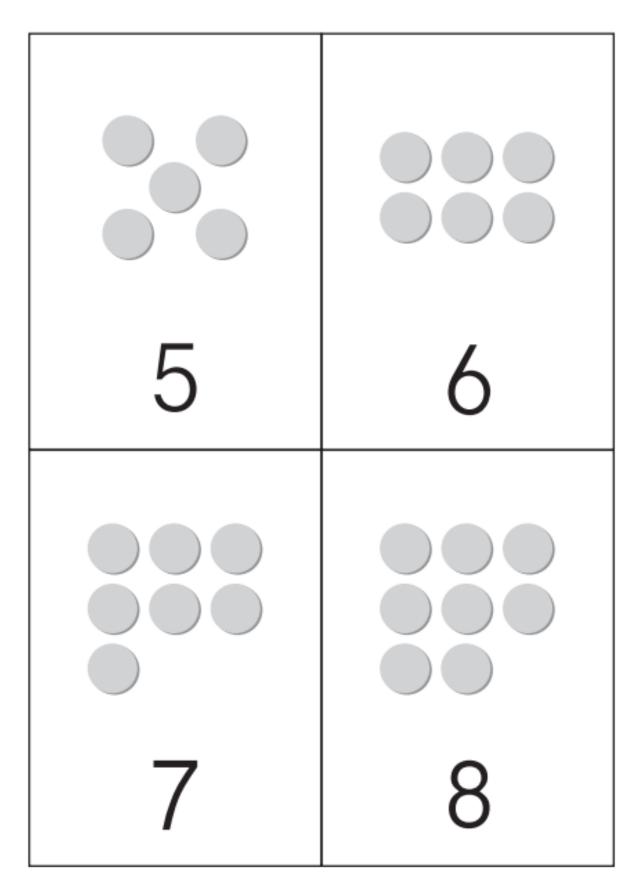
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TR<sub>2</sub>

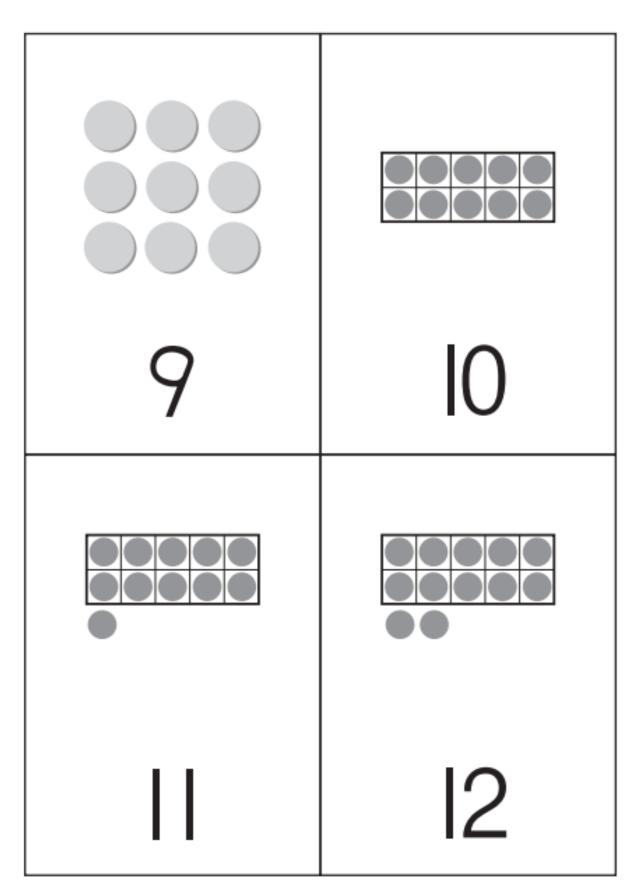
Counters and Numerals (1-4)



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TR3

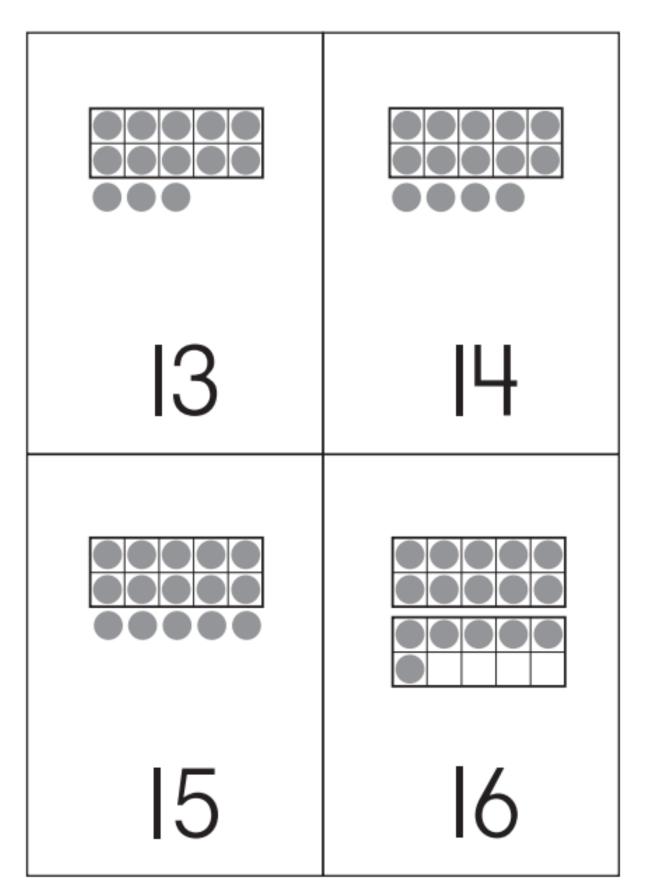
Counters and Numerals (5-8)



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TR4

Counters and Numerals (9-12)



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TR5

Counters and Numerals (13-16)

